Q1 Summative Review
Secondary 3H

Name: Answer Key
Date: _____ Class: _____

Unit 1
Transformations of Radical Graphs
1-1a: I know the parent function graphs

1. (no calc) Given the parent function $f(x)$, write the equation that contains the given transformations.

a. $f(x)=\sqrt[3]{x}$

- Vertical Translation down two units
- Horizontal reflection

$f(x) = \sqrt[3]{-x} - 2$

b. $f(x)=\sqrt{x}$

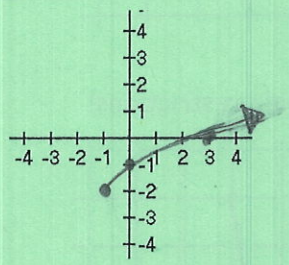
- Vertical Stretch by a factor of 3
- Vertical reflection

$f(x) = -3\sqrt{x}$

Attributes of Radical Functions/Transformations of Radical Graphs
1-1c: I can graph square and cube root functions with and without transformations
1-1b: I can analyze key attributes of a graph

(no calc) Graph the following functions **without** using a calculator. Next, identify the parent function, list the transformations involved, and also include the new domain and range.

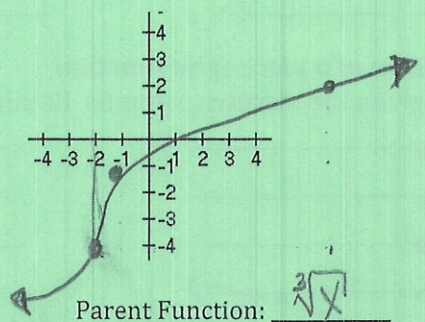
2. $g(x)=\sqrt{x+1}-2$



Parent Function: $\sqrt{x}$
List the transformations in words:
a) 1 unit left
b) 2 units down

Domain: $[-1,\infty)$
Range: $[-2,\infty)$
x-int: $(3,0)$
y-int: $(0,-1)$
Left EB: N/A
Right EB: $\lim_{x\to\infty} g(x) = \infty$
Inc: $(-1,\infty)$
Dec: N/A
Max: N/A
Min: $(-1,-2)$

3. $f(x)=3\sqrt[3]{x+2}-4$



Parent Function: $\sqrt[3]{x}$
List the transformations in words:
a) Vertical Stretch 3
b) 2 units left
c) 4 units down

Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$
x-int: $(\sqrt[3]{\tfrac{4}{3}}-2,0)$ or $(1.1,0)$
y-int: $(0,-.22)$
Left EB: $\lim_{x\to-\infty} f(x) = -\infty$
Right EB: $\lim_{x\to\infty} f(x) = \infty$
Inc: $(-\infty,\infty)$
Dec: N/A
Max: N/A
Min: N/A

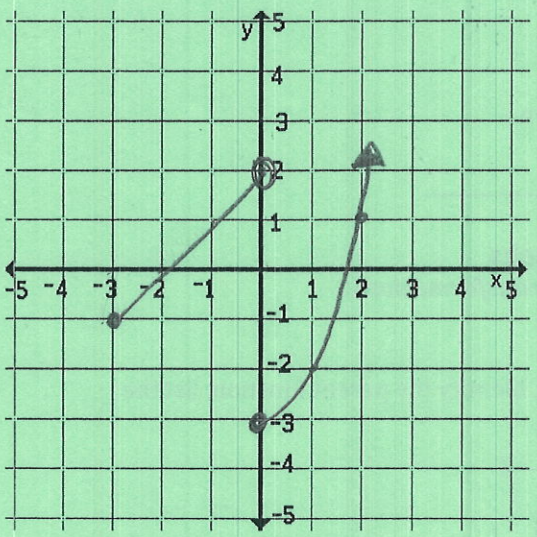
x-intercept
$0 = 3\sqrt[3]{x+2} - 4$
$3\left(\frac{4}{3}\right) = 3\sqrt[3]{x+2}$
$y = 3\sqrt[3]{0+2} - 4$
$= 3\sqrt[3]{2} - 4$

Piece-wise Graphs

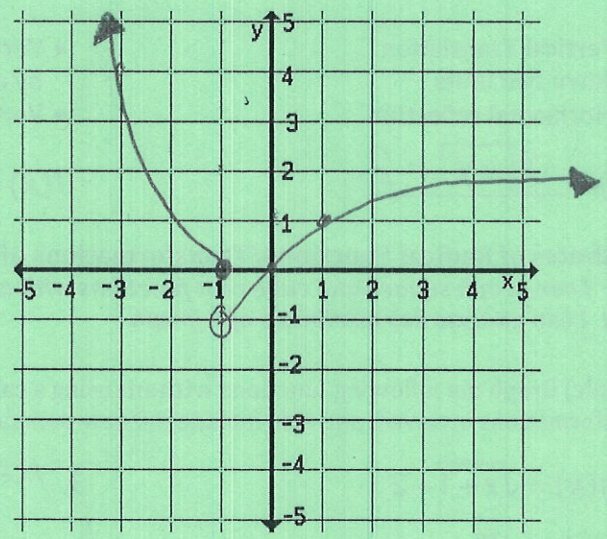
1-3a: I can graph a piece-wise function

(no calc) Graph the piece-wise functions

4. $f(x)=\begin{cases} x+2, & -3\le x<0 \\ x^2-3, & x\ge 0 \end{cases}$



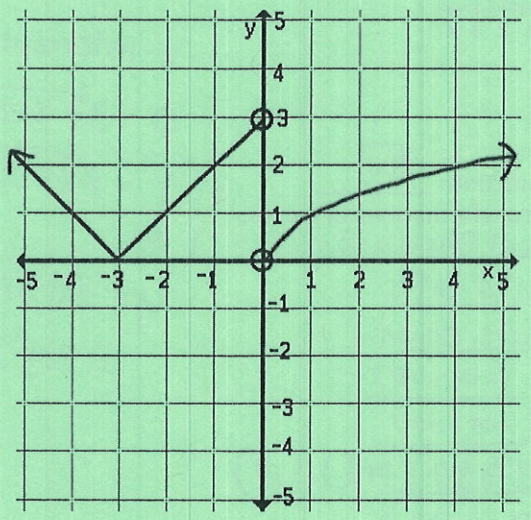
5. $f(x)=\begin{cases} (x+1)^2, & x\le -1 \\ \sqrt[3]{x}, & x>-1 \end{cases}$



1-3b: I can write an equation of a piece-wise function

(no calc) Write a function given the piecewise graphs. Be sure to include any domain restrictions!

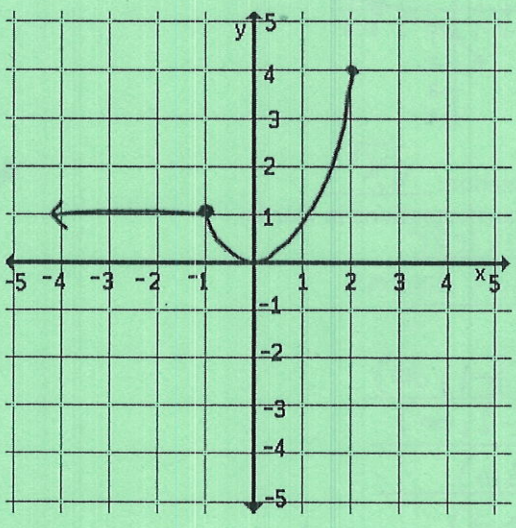
6.



Function:

$f(x)\begin{cases} |x+3|, & x<0 \\ \sqrt{x}, & x>0 \end{cases}$

7.



Function:

$f(x)=\begin{cases} 1, & x\le -1 \\ x^2, & -1<x\le 2 \end{cases}$

Solving Radical Equations

1-2a: I can solve radical equations and check for extraneous solutions.

8. (calc allowed) Solve for x. Check for extraneous solutions. $(x-6)^2=(\sqrt{18-3x})^2$

$x^2-12x+36=18-3x$

$+3x \quad -18 \quad -18 \quad +3x$

$x^2-9x+18=0$

$(x-6)(x-3)=0$ $x=6$ $x=3$ extraneous

$0=\sqrt{0}$ ✓ $x=6$

$3-6=\sqrt{18-9}$

$-3 \neq 3$

9. (calc allowed) Solve for x. $2\sqrt[3]{5x+1}-2=10$

$+2 \quad +2$

$\frac{2}{2}$

$(\sqrt{5x+1})^2=(6)^2$

$5x+1=36$

$-1 \quad -1$

$5x=35$

$x=7$

$2\sqrt{36}-2=10$

$10=10$ ✓

Unit 2

Series

Find the stated term for the following sequences

1. (calc allowed) −3, −6, −12, −24, ...; 9th term $a=-3$ $r=2$

×2 ×2

$f(9)=(-3)(2)^{9-1}=-768$

2. (calc allowed) A geometric sequence that has a first term 2, ends with -4374 and has a common ratio of -3, how many terms are in the sequence?

$a=2$ $f(n)=-4374$ $r=-3$ find n

$\frac{-4374}{2}=\frac{2}{2}(-3)^{n-1}$

$-2187=(-3)^{n-1}$

$(-3)^7=(-3)^{n-1}$

$7=n-1$

$8=n$

Graphing Exponentials

3. (no calc) What is the y intercept of $y=6\left(\frac{1}{2}\right)^x$? let $x=0$

$y=6\left(\frac{1}{2}\right)^0 \rightarrow y=6(1) \Rightarrow y=6$ $(0,6)$

4. (no calc) For the function $f(x)=5^x$, what is the limit as $x\to\infty$?

A. $\lim_{x\to\infty} f(x)=\infty$ B. $\lim_{x\to\infty} f(x)=-\infty$ C. $\lim_{x\to\infty} f(x)=0$ D. $\lim_{x\to\infty} f(x)=1$

(no calc) Find the domain, range, and y-intercept for the following functions without graphing.

5. $f(x)=2(3)^{x-2}-1$

Domain: $(-\infty,\infty)$

Range: $(-1,\infty)$

y-int: $(0,-7/9)$

Asymptote: $y=-1$

$2(3)^{(0-2)}-1$

$2(3)^{-2}-1$

$\frac{2}{9}-1 \rightarrow \frac{2}{9}-\frac{9}{9}=\frac{-7}{9}$

6. $f(x)=\left(\frac{1}{3}\right)^x+2$

Domain: $(-\infty,\infty)$

Range: $(2,\infty)$

y-int: $(0,3)$

Asymptote: $y=2$

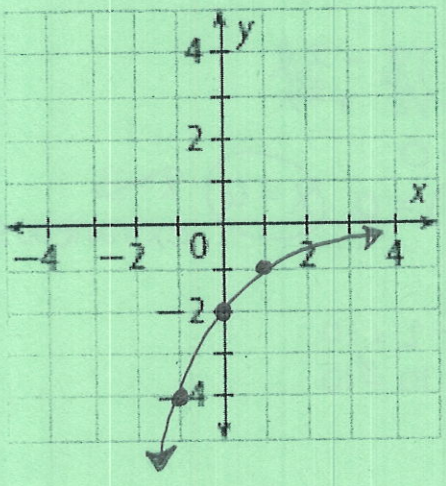
$\left(\frac{1}{3}\right)^0+2$

$=1+2$

$=3$

(no calc) Graph the following without a calculator and label any asymptotes or intercepts

7. $g(x) = -2\left(\frac{1}{2}\right)^x$

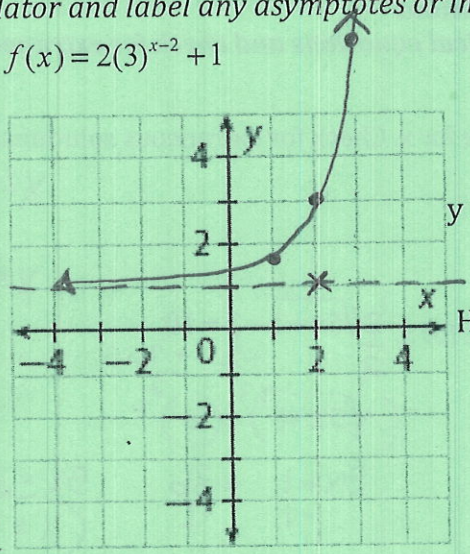


y-int: (0, -2)

HA $y = 0$

x	$(\frac{1}{2})^x$	
-1	2^{x-2}	-4
0	1^{x-2}	-2
1	$\frac{1}{2}^{x-2}$	-1

8. $f(x) = 2(3)^{x-2} + 1$



y int (0, 1 2/9)

HA _____

$2(3)^{0-2} + 1$

$2(3)^{-2} + 1$

$\frac{2}{9} + 1$

x	3^x	
-1	$\frac{1}{3} \times 2$	2/3
0	1×2	2
1	3×2	6

Exponential Functions

9. (calc allowed) If Jane invests $4,200 at an 8% interest **compounded continuously**, how much money will there be after 10 years?

$A(10) = 4200e^{.08 \cdot 10}$ = $9347.27

(10-12) Answer the following questions with the following: an investment of $2000 that earns 3.4% interest

10. (calc allowed) Write an equation to describe the value A(t) of the investment at time t if the interest is compounded annually.

$$A(t) = 2000\left(1 + \frac{.034}{1}\right)^{1 \cdot t}$$

11. (calc allowed) What is the value of the investment after 10 years?

$$A(10) = 2000\left(1 + \frac{.034}{1}\right)^{1 \cdot 10} = \$2794.06$$

12. (calc allowed) How long would it take for the investment to reach $10,000?

$$10000 = 2000\left(1 + \frac{.034}{1}\right)^{1 \cdot t}$$

$$5 = (1.034)^t \qquad \log_{(1.034)} 5 = t$$

$t = 48.1$ years

13. (calc allowed) A melting snowman is losing one-half of his weight each day. He originally weighed 128 pounds. Assuming that the outside temperature stays the same, how much does the snowman weigh after 5 days?

$a = 128$

$r = 1/2$

$t = 5$

$S(5) = 128\left(\frac{1}{2}\right)^5$

= 4 lbs after 5 days.

14. (calc allowed) A car with a cost of \$25,000 is decreasing in value at a rate of 10% each year. The function $g(t) = 25{,}000(0.9)^t$ gives the value of the car after t years. When will the value of the car be about \$12,000?

$12000 = 25000(0.9)^t$

$.48 = (0.9)^t$

$\log_{0.9} .48 = t \quad \Rightarrow t = 7.0$ years.

15. (calc allowed) An online video game tournament begins with 4096 players. Four players play in each game. In each game, there is only one winner, and only the winner advances to the next round. How many games will the winner play?

$V(g) = 4096\left(\frac{1}{4}\right)^g$

$1 = 4096\left(\frac{1}{4}\right)^g \Rightarrow \frac{1}{4096} = \frac{1}{4}^g$

$\log_{\frac{1}{4}}\left(\frac{1}{4096}\right) = g \Rightarrow g = 6$

1024, 256, 64, 16, 4, 1

* The winner will need to play 6 games.

Unit 3

Evaluating Logarithms

(no calc) Write the given exponential equation as a logarithmic equation

1. $4^2 = 16$ → $\log_4 16 = 2$
2. $e^{17} = a$ → $\ln a = 17$
3. $10^4 = 10{,}000$ → $\log 10000 = 4$
4. $b^p = a$ → $\log_b a = p$

(no calc) Write the given logarithmic equation as an exponential equation

5. $\log_7 x = 10$ → $7^{10} = x$
6. $\ln x = 32$ → $e^{32} = x$
7. $\log 1000 = 3$ → $10^3 = 1000$
8. $\log_\Delta \Phi = \Psi$ → $\Delta^\Psi = \Phi$

9. (no calc) Evaluate without using a calculator:

A. $\log_5 125 = (3)$ → $5^3 = 125$

B. $\log_5 \frac{1}{25} = (-2)$ → $5^{-2} = \frac{1}{25}$

C. $\log_5 \sqrt{5} = \left(\frac{1}{2}\right)$ → $5^{1/2} = \sqrt{5}$

Logarithm Properties

(no calc) Evaluate the following without a calculator:

10. $\log_4 1 = (0)$ → $4^{\square} = 1$
11. $\ln e = (1)$ → $e^{\square} = e$
12. $\log_5 5 = (1)$ → $5^{\square} = 5$
13. $7^{\log_7 12} = (12)$
14. $\log_{12} 12^{15} = (15)$
15. $\ln e^{32} = (32)$ → $e^{\square} = e^{32}$
16. $10^{\log 14} = 14$
17. $\log_7 \sqrt[3]{7} = \left(\frac{1}{3}\right)$ → $7^{\square} = 7^{\frac{1}{3}}$

(no calc) Write each as a single logarithm.

18. $3\log_4 2+\log_4 6$

$= \log_4 2^3 + \log_4 6$

$= \log_4(8\cdot 6) = \boxed{\log_4 48}$

19. $\frac{1}{3}\log_7 y - 6\log_7 z$

$= \log_7 y^{\frac{1}{3}} - \log_7 z^6$

$= \boxed{\log_7 \frac{\sqrt[3]{y}}{z^6}}$

20. $(3\log_2 x+\frac{1}{2}\log_2 y)-2\log_2(xz)$

$\log_2 x^3 + \log_2 \sqrt{y} - \log_2 x^2 z^2$

$\boxed{\log_2 \frac{x^3\sqrt{y}}{x^2 z^2}}$

(no calc) Use the properties of logarithms to expand the following. Express all exponents as coefficients.

21. $\log_3 x^2 y^4$

$2\log_3 x + 4\log y$

22. $\log_{12}\frac{\sqrt{x}}{y^2}$

$= \frac{1}{2}\log_{12} x - 2\log y$

23. $\log_4 \frac{x\sqrt{y}}{z^{12}w^2}$

$= (\log x + \frac{1}{2}\log y) - (12\log z + 2\log w)$

(calc allowed) Evaluate the following logarithms.

24. $\log_5 3 = (.68)$

25. $\log 80000 = (4.9)$

Solving Logarithms

(no calc) Solve the following. Check for extraneous solutions.

26. $4^{2x+10}+6=70$

$-6 \quad -6$

$4^{2x+10} = 64$

$4^{2x+10} = 4^3$

$2x+10 = 3$

$-10 \quad -10$

$2x = -7$

$x = -7/2$

27. $e^{\frac{x}{4}}+6=7$

$-6 \quad -6$

$e^{x/4} = 1$

$\ln 1 = \frac{x}{4}$

$0 = \frac{x}{4}$

$0 = x$

28. $\log_2 x - \log_2 3 = 4$

$\log_2 \frac{x}{3} = 4$

$2^4 = \frac{x}{3}$

$3\cdot 16 = \frac{x}{3}\cdot 3$

$48 = x$

29. $\ln(x-1)+8=8$

$\ln(x-1) = 0$

$e^0 = x-1$

$1 = x-1$

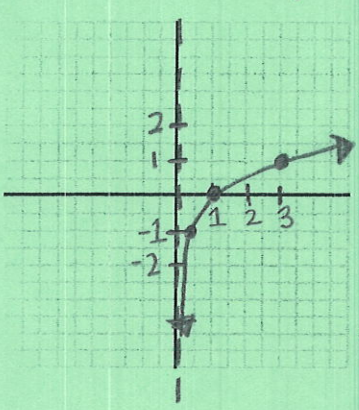
$2 = x$

Graphing Logarithms

(no calc) Without a calculator, graph the following, list the transformations (if any), asymptote and two points:

30. $f(x)=\log_3 x$

1/3	-1
1	0
3	1



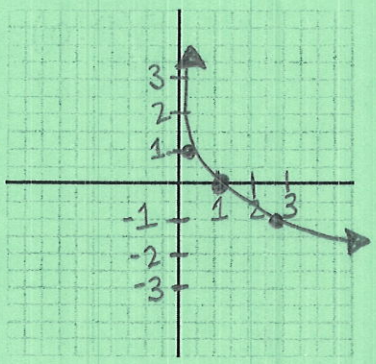
Transformations: none

Points: (1,0)(3,1)

Asymptote: $x=0$

31. $f(x)=-\ln x$

1/e	-1	-1	1
1	0	-1	0
e	1	-1	-1



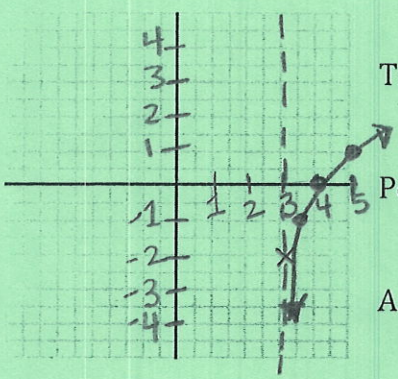
Transformations: Vertical flip

Points: (1,0)(e,-1)

Asymptote: $x=0$

32. $f(x)=\log_2(x-3)-2$

2^x	
1/2	-1
1	0
2	1

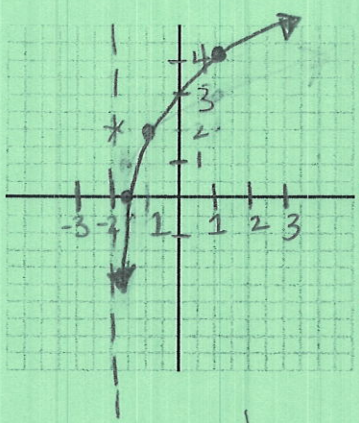


Transformations: 3 units right 2 down

Points: (4,0)(5,1)

Asymptote: $x=3$

33. $f(x)=2\log_3(x+2)+2$



Transformations: Vertical Stretch 2 2 units Left 2 units up

Points:

Asymptote: $x=-2$

1/3	-1 x2	-2
1	0 x2	0
3	1 x2	2