Secondary Math 3H Quarter 3 Final Review Name: Answer Key

Class: 1

Polynomial Operations

1. Write the polynomial $-23x^7 + x^9 - 6x^3 + 10 + 2x^2$ in standard form, and then identify the degree and leading coefficient.

$x^9 - 23x^7 - 6x^3 + 2x^2 + 10$ degree 9 leading coefficient 1

Perform the indicated operation.

2. $(17x^8 + 11x^2 - 6) + (18x + 7x^8 - 12x^2 + 3)$

$= 7x^8 - 12x^2 + 3\ 18x$

$= 7x^8 - x^2 + 18x - 3$

3. $(-2x + 13x^5 + 11) - (5 - 9x^3 + x)$

$-x \quad -5 \quad +9x^3$

$13x^5 + 9x^3 - 3x + 6$

4. $(9x^2 - x + 4) - (5x + 7) + (6x - 11)$

$-5x \quad -7 \quad 6x \quad 11$

$9x^2 + 8$

5. $(15x - 11x^{12} + x^9 - x^7 + 3x^2) + (x^7 - 6x^2 - x^9)$

$-x^9 \quad x^7 \quad -6x^2$

$-11x^{12} - 3x^2 + 15x$

6. $(2x + 3y)(3x^2 - xy + 2y^2)$

	$3x^2$	$-xy$	$2y^2$
$2x$	$6x^3$	$-2x^2y$	$4xy^2$
$3y$	$9x^2y$	$-3xy^2$	$6y^3$

$6x^3 + 7x^2y + xy^2 + 6y^3$

7. $(x^3 + 25x^2 + 100x) \div (x + 20)$

-20 | 1 25 100 0

↓ −20 100 0

1 5 0 0

$x^2 + 5x$

8. $(7x^3 - 4x^2 - 400x - 100) \div (x - 8)$

8 | 7 4 −400 −100

↓ 56 480 160

7 60 80 60

$x^2 + 60x + 80 + \frac{60}{x-8}$

9. $(3x^3 - 11x^2 - 56x - 50) \div (x + 4)$

-4 | 3 −11 −56 −50

↓ −12 92 −144

3 −23 36 −194

$3x^2 - 23x + 36 - \frac{194}{x+4}$

10. $(x^3 + 8x^2 + 13x + 2) \div (x + 2)$

-2 | 1 8 13 2

↓ −2 −18 12

1 6 −6 14

$x^2 + 6x - 6 + \frac{14}{x+2}$

11. $(5x^3 + 4x^2 - 4x + 1) \div (x + 1)$

-1 | 5 4 −4 1

↓ −5 2 2

5 −1 −2 3

$5x^2 - x - 2 + \frac{3}{x+1}$

Binomial Theorem
Expand the following binomials.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

12. $(x-5)^4$

$1x^4 + 4x^3(-5)^1 + 6x^2(-5)^2 + 4x(-5)^3 + 1(-5)^4$

$x^4 - 20x^3 + 150x^2 - 500x^3 + 625$

13. $(x+y)^6$

$x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

14. $(2x-1)^3$

$1(2x)^3 + 3(2x)^2(-1) + 3(2x)(-1)^2 + (-1)^3$

$8x^3 - 12x^2 + 6x - 1$

15. $(3x+y)^5$

$(3x)^5 + 5(3x)^4y + 10(3x)^3y^2 + 10(3x)^2y^3 + 5(3x)y^4 + y^5$

$243x^5 + 405x^4y + 270x^3y^2 + 90x^2y^3 + 15xy^4 + y^5$

16. Find the 15th term of the following binomial: $(x-1)^{16}$

$\binom{16}{14}x^2(-1)^{14} = \binom{16}{14}x^2 =$

17. Find the 5th term of the following binomial: $(x-2)^7$

$35x^3(-2)^5 = -1120x^3$

18. Find the 3rd term of the following binomial: $(x+2)^5$

$10x^3(2)^2 = 40x^3$

19. Find the 2nd term of the following binomial: $(2x-1)^4$

$4(2x)^3(-1)^1 = -32x^3$

Factoring Polynomials: Factor each polynomial completely.

20. $p^2 - 3p - 40$

$(p-8)(p+5)$

$-8p$

$5p$

$-3p$ ✓

21. $3x^2 - x - 4$

$(3x-4)(x+1)$

$-4x$

$+3x$

$-x$ ✓

22. $(3b^3 - 6b^2) + (2b - 4)$ — factor by grouping

$3b^2(b-2) + 2(b-2)$

$(b-2)(3b^2+2)$

$a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$

difference of Cubes

23. $2k^2-5k+2$

$(2k-1)(k-2)$

$-1k$

$-4k$

$-5k$ ✓

24. b^2+4b

$b(b+4)$

25. $2x^3-2$

$2(x^3-1)$

$2(x-1)(x^2+x+1)$

26. $4n^4+68n^3+280n^2$

$4n^2(n^2+17n+70)$

$4n^2(n+10)(n+7)$

$10n$

$7n$ $17n$ ✓

27. $5x^2+x-4$

$(5x-4)(x+1)$

$-4x$

$+5x$ x ✓

28. $64a^3-27$ $4a\ 4a\ 4a$ $3\ 3\ 3$

$(4a-3)(16a^2+12a+9)$

29. n^2+6n+8

$(n+4)(n+2)$

$4n$

$2n$ $6n$ ✓

30. $25x^2-9$ Difference of Squ.

$(5x-3)(5x+3)$

$-15x$

$15x$

$0x$ ✓

31. $3x^2-x-2$

$(3x+2)(x-1)$

$+2x$

$-3x$ $-x$ ✓

32. x^2-4

$(x-2)(x+2)$

33. $27x^3+1$

$(3x+1)(9x^2-3x+1)$

34. x^3+125

$(x+5)(x^2-5x+25)$

35. $v^2-11v+24$

$(v-8)(v-3)$

$-8v$

$-3v$

$-11v$ ✓

36. $10p^3+10p^2+15p+15$

$10p^2(p+1)+15(p+1)$

$(p+1)(10p^2+15)$

$5(p+1)(2p^2+3)$

Zeros of Polynomials

37. Are (x+2) and (x-6) factors of $f(x) = 2x^3 + 8x^2 - 22x - 60$?

-2 | 2 8 −22 −60

↓ −4 −8 60

2 4 −30 0

yes $(x+2)$ ↑ remainder Zero

6 | 2 8 −22 −60

↓ 12 80 348

2 20 58 288

$(x-6)$ is not a factor.

Not a zero remainder

Find **all** the zeros of the following functions **WITHOUT** a calculator

38. $g(x) = x^3 + 4x^2 + 4x$

$x(x^2+4x+4)$

$x(x+2)^2$

$x = 0$

$x = -2$ multiplicity 2

39. $h(x) = 3x^3 - 2x^2 - 3x + 2$

$x^2(3x-2) - (3x-2)$

$(3x-2)(x^2-1)$

$(3x-2)(x-1)(x+1)$

$x = \frac{2}{3}, 1, -1$

40. $f(x) = x^4 + x^3 - 14x^2 - 2x + 24$

4 | 1 1 -14 -2 24

↓

1

41. $k(x) = 7x^3 + x^2 - 28x - 4$

$= x^2(7x+1) - 4(7x+1)$

$= (7x+1)(x-2)(x+2)$

Zero: $-\frac{1}{7}, 2, -2$

Given the following zeros and multiplicities, write a function in factored form

42. 2 (multiplicity of 3), 5, –7(multiplicity of 2)

$(x-2)^3(x-5)(x+7)^2$

43. 4, 2(multiplicity of 5), -3

$(x-4)(x-2)^5(x+3)$

44. Given $g(x) = 3x^3 - 8x^2 + 3x + 2$, use the rational root theorem to determine which of the following are ***possible zeros*** of the function.

a. 2 b. -3 c. 4 d. $-\frac{2}{3}$ e. $\frac{3}{4}$

$\frac{2}{}: 1, 2$

$\frac{}{3}: 1, 3$

Possible

$\pm 1, \pm\frac{1}{3}, \pm 2, \pm\frac{2}{3}$

Graphing from Factored Form

For the following functions, find the zeros, state the end behavior using limit notation, and graph the function.

45. $f(x) = -(x+2)^2(x-1)$

$-4(-1)$

neg cubic

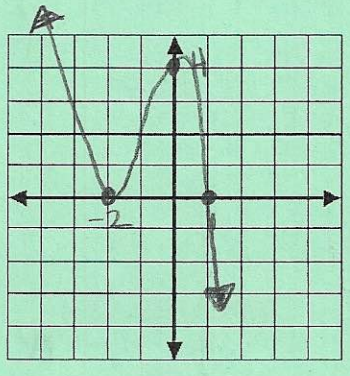
$y =$

zeros: $-2, 1$

mult. 2

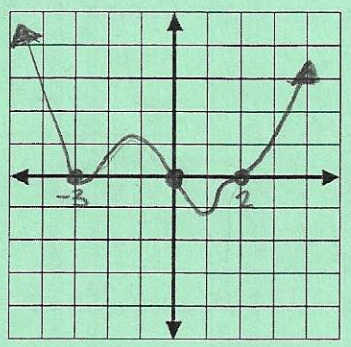
End Behavior:

$\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = -\infty$



pos. 6th degree

46. $h(x) = x(x+3)^2(x-2)^3$



zeros: $0, -3$ (mult 2) 2 (mult 3)

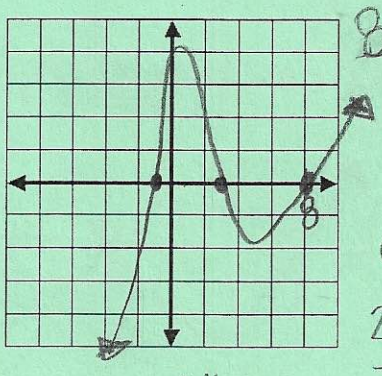
End Behavior:

$\lim_{x \to -\infty} h(x) = \infty$ $\lim_{x \to \infty} h(x) = \infty$

Graphing from Standard Form

47. $f(x) = x^3 - 10x^2 + 14x + 16$

positive cubic



zeros: $8, 1 \pm \sqrt{3}$

8 | 1 −10 14 16
↓ 8 −16 −16
1 −2 −2 0

$x^2 - 2x - 2$

$a = 1 \quad b = -2 \quad c = -2$

$\frac{2 \pm \sqrt{(4) - 4(1)(-2)}}{2}$

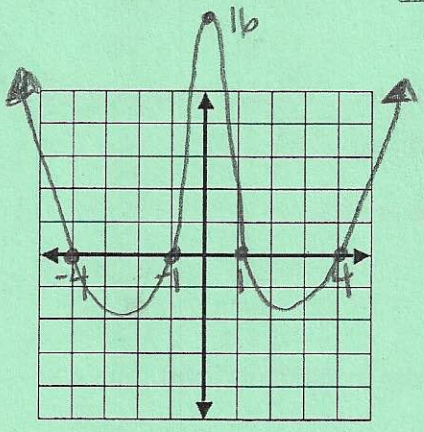
$\frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$

between 1 and 2

End Behavior:

$\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = \infty$

48. $g(x) = x^4 - 17x^2 + 16$



$(x^2 - 16)(x^2 - 1)$

$(x-4)(x+4)(x+1)(x-1)$

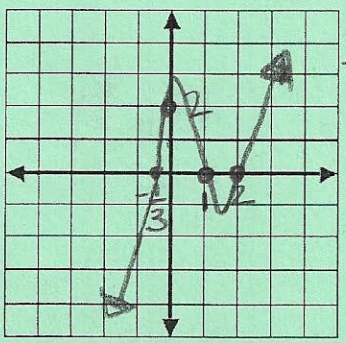
zeros: $\pm 4, \pm 1$

End Behavior:

$\lim_{x \to -\infty} g(x) = \infty$ $\lim_{x \to \infty} g(x) = \infty$

49. $f(x) = 3x^3 - 8x^2 + 3x + 2$

positive cubic



zeros: $1, -\frac{1}{3}, 2$

1 | 3 −8 3 2
↓ 3 −5 −2
3 −5 −2 0

$3x^2 - 5x - 2$

$(3x + 1)(x - 2)$

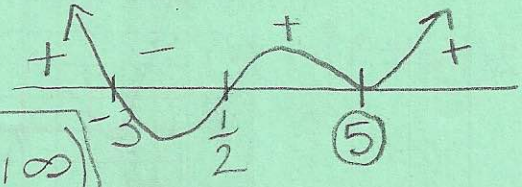
End Behavior:

$\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = \infty$

Solving Inequalities

Solve the inequality using a sign chart. Complete factoring if needed. Write your answer in interval notation.

50. $x(x-5)^2(x+3)(2x-1) \geq 0$



$(-\infty, -3] \cup [\frac{1}{2}, \infty)$

4th degree positive

51. $x^4 - 5x^3 - 4x^2 + 44x - 48 > 0$, given 2 is a zero *(multiplicity 2)*

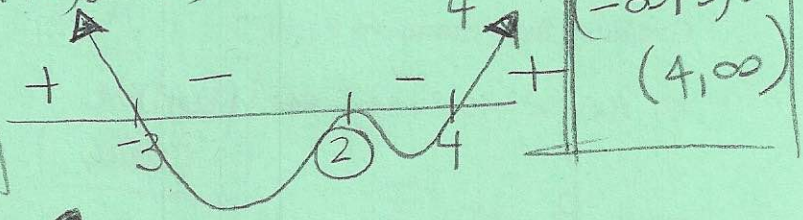
4th degree positive

2	1	−5	−4	44	−48
	↓	2	−6	−20	48
2	1	−3	−10	24	0
	↓	2	−2	−24	
	1	−1	−12	0	

$x^2 - x - 12$

$(x+3)(x-4)$

Zeros: 2 (mult 2), −3, 4



$(-\infty, -3) \cup (4, \infty)$

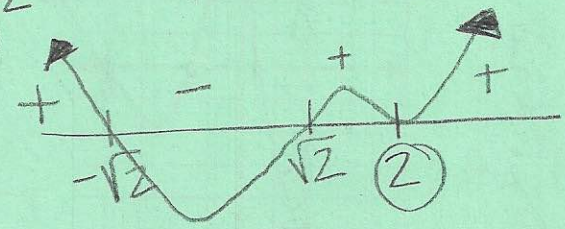
52. $(x^2-2)(x^2-4x+4) \leq 0$

pos. 4th deg.

$(x^2-2)(x-2)^2$

$\sqrt{x^2} = \sqrt{2}$

$x = \pm\sqrt{2}$



$[-\sqrt{2}, \sqrt{2}] \cup [2]$

Complex Numbers

Add or subtract as indicated. Write answer in standard form.

53. $(-3+\sqrt{-25})+(4-\sqrt{-16})$

$-3+5i+4-4i$

$1+i$

54. $(-2+\sqrt{-81})+(-4-\sqrt{-64})$

$-2+9i-4-8i$

$-6+i$

Multiply. Write answers in standard form.

55. $5i(2-5i)$

$10i - 25i^2 = 25+10i$

56. $(2-3i)(4+2i)$

$8+4i-12i-6i^2$

$8-8i+6$

$14-8i$

Simplify

57. $\frac{2}{4i}\,\frac{i}{i} = \frac{2i}{4i^2} = \frac{2i}{-4} = -\frac{i}{2}$

58. $\frac{3-4i}{2+i}\,\frac{2-i}{2-i} = \frac{6-3i-8i+4i^2}{4+1}$

$= \frac{2-11i}{5}$

Complex Zeros

$x^2+9=0$
$\sqrt{x^2}=\sqrt{-9}$
$x=\pm 3i$

Find all the zeros of $f(x)$. Include any multiplicities greater than 1. Classify the roots as real or imaginary.

59. $f(x)=2x^3-3x^2+4x-6$

$x^2(2x-3)+2(2x-3)$

$(2x-3)(x^2+2)$

$x^2+2=0$
$\sqrt{x^2}=\sqrt{-2}$
$x=\pm i\sqrt{2}$

Zeros: Real: $\frac{3}{2}$
Imaginary: $\pm i\sqrt{2}$

60. $f(x)=x^4-81$

$(x^2-9)(x^2+9)$

$(x-3)(x+3)(x^2+9)$

Zeros: Real: ± 3
Imaginary: $\pm 3i$

Given the zero, find the remaining zeros.

61. $4i,\ f(x)=x^4+13x^2-48$

1 48
2 24
3 16

$(x^2+16)(x-3)$

$x^2+16=0$
$x=\pm 4i$

Remaining Zeros: $3, -4i$

62. $3i, f(x)=x^3+x^2+9x+9$

$x^2(x+1)+9(x+1)$

$(x+1)(x^2+9)$

Remaining Zeros: -1, $-3i$

63. $-2i, f(x)=x^4-3x^3-6x^2-12x-40$

$-2i$	1	-3	-6	-12	-40
	↓	$-2i$	$-4+6i$	$+12-20i$	40
$2i$	1	$-3-2i$	$-10+6i$	$+20i$	0
	↓	$+2i$	$-6i$	$-20i$	
	1	-3	-10	0	

$x^2-3x-10$

$(x-5)(x+2)$

Remaining: $2i, 5, -2$ Zero