HW 9-3 Name: Answer Key

Finding Complex Zeros and Writing Polynomials from all Zeros

Find all the zeros of f(x). Include any multiplicities greater than 1. Classify the roots as real or imaginary.

1. $f(x) = 2x^5 + 3x^3 - 5x$

$= x(2x^4 + 3x^2 - 5)$

$= x(2x^2 + 5)(x^2 - 1)$

$= x(2x^2 + 5)(x-1)(x+1)$

$2x^2 + 5 = 0$

$2x^2 = -5$

$x^2 = -\frac{5}{2}$

$x = \pm i\frac{\sqrt{10}}{2}$

Real Zeros: 0, 1, -1

Imaginary: $\pm i\frac{\sqrt{10}}{2}$

2. $f(x) = x^3 - 3x^2 + 4x - 12$

$x^2(x-3) + 4(x-3)$

$(x-3)(x^2+4)$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

Real Zeros: 3

Imaginary: $\pm 2i$

Solve the polynomial equation by finding all roots.

3. $2x^3 - 3x^2 + 8x - 12 = 0$

$x^2(2x-3) + 4(2x-3) = 0$

$(2x-3)(x^2+4) = 0$

$x^2 + 4 = 0$

$x = \pm 2i$

Real Zero: $\frac{3}{2}$

Imaginary: $\pm 2i$

4. $x^4 - 5x^3 + 3x^2 + x = 0$

$x(x^3 - 5x^2 + 3x + 1)$

+1	1	-5	3	1
	↓	1	-4	-1
	1	-4	-1	0

$x^2 - 4x - 1$

$x = \frac{4 \pm \sqrt{16 - 4(-1)}}{2}$

$= \frac{4 \pm \sqrt{20}}{2}$

$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

Real Zeros: 1, $2 \pm \sqrt{5}$

Imaginary: n/a

5. Given the zero 2i find the remaining zeros for the polynomial $f(x) = x^4 - 16$

$x^4 - 16 = (x^2 - 4)(x^2 + 4)$

$= (x-2)(x+2)(x^2+4)$

Remaining zeros: 2, -2, 2i, -2i

6. Given the zero -3i find the remaing zeros for the polynomial $f(x) = x^4 - 2x^3 + x^2 - 18x - 72$

$(x - 3i)(x + 3i)$

$x^2 + 9$

$x^2 - 2x - 8 \rightarrow (x-4)(x+2)$

$x^2+9 \;\overline{)\; x^4 - 2x^3 + x^2 - 18x - 72}$

$-x^4 \quad +9x^2$

$-2x^3 - 8x^2 - 18x - 72$

$+2x^3 \quad +18x$

$-8x^2 - 72$

0

Real Zeros: 4, -2

Imaginary: $\pm 3i$

HW 9-3 Name: _____

Write the polynomial function in standard form with least degree and a leading coefficient of 1 that has the given zeros.

7. $0, \sqrt{5} \text{ and } 2$

$x=0 \quad x=\sqrt{5} \quad x=2$

$x(x-\sqrt{5})(x-2)$

$(x^2-2x)(x-\sqrt{5})$

$x^3-\sqrt{5}x^2-2x^2+2\sqrt{5}x$

8. $4i, 2 \text{ and } -2$

$(x-2)(x+2)(x-4i)(x+4i)$

$(x^2-4)(x^2+16)$

$x^4+16x^2-4x^2-64$

x^4+12x^2-64

$x-(3-2i)$

$x\ (3+2i)$

9. $1, -1 (\text{multiplicity } 3), \text{ and } 3i$

$(x-1)(x+1)^3(x-3i)(x+3i)$

$(x-1)(x+1)^3(x^2+9)$

10. $2 \text{ and } 3-2i$ $3+2i$

$(x-2)(x-3+2i)(x-3-2i)$

$(x-2)(x^2-6x+13)$

x^3-6x^2+13x

$-2x^2+12x-26$

$x^3-8x^2+25x-26$

11. **Critical Thinking** What is the least degree of a polynomial equation that has $3i$ as a root with a multiplicity of 3, and $2-\sqrt{3}$ as a root with multiplicity 2? Explain.

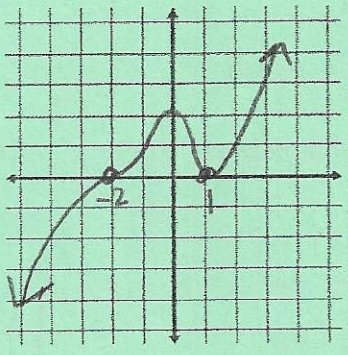
Zeros: $3i\ 3i\ 3i, -3i, -3i, -3i, 2-\sqrt{3}, 2-\sqrt{3}$

degree 8

Review

1. Graph the following functions and state the domain, range, and end behavior

$f(x)=(x+2)^3(x-1)^2$



Zeros and Multiplicity:

-2 mult 3

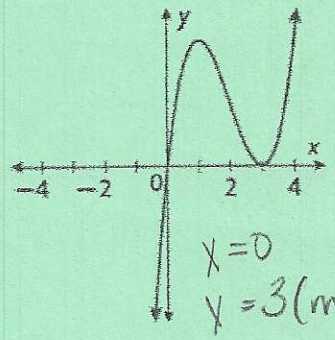
1 mult 2

End Behavior:

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = \infty$

2. Write a function for the following graph



$x=0$

$y=3 (\text{mult } 2)$

$x(x-3)^2$