

Zeros of a Polynomial

Determine whether the given binomial is a factor of the polynomial  $p(x)$ . If so, find the remaining factors of  $p(x)$ .

12.  $p(x) = x^3 + 2x^2 - x - 2, (x+2)$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -1 & -2 \\ & \downarrow & -2 & 0 & 2 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

(yes)

$(x+2)(x+1)(x-1)$

13.  $p(x) = 2x^4 + 6x^3 - 5x - 10, (x+2)$

$$\begin{array}{r|rrrrr} -2 & 2 & 6 & 0 & -5 & -10 \\ & \downarrow & -4 & -4 & 8 & -6 \\ \hline & 2 & 2 & -4 & 3 & -16 \end{array}$$

not a factor

14.  $p(x) = x^3 - 22x^2 + 157x - 360, (x-8)$       15.  $p(x) = 4x^3 - 12x^2 + 2x - 5, (x-3)$

8 | 1 -22 157 -360

$$\begin{array}{r|rrrr} 8 & 1 & -22 & 157 & -360 \\ & \downarrow & 8 & -112 & 360 \\ \hline & 1 & -14 & 45 & 0 \end{array}$$

(yes)

$(x-8)(x-5)(x-9)$

3 | 4 -12 2 -5

$$\begin{array}{r|rrrr} 3 & 4 & -12 & 2 & -5 \\ & \downarrow & 12 & 0 & 6 \\ \hline & 4 & 0 & 2 & 1 \end{array}$$

not a factor

Find all the zeros of the following polynomials GRAPHICALLY (using your calculator)

16.  $f(x) = x^5 + 2x^4 - 2x^2 - x$

Zeros are  
 $x = -1, 0, 1$

17.  $f(x) = x^4 - 4x^3 + 5x^2 - 2x$

Zeros are:  
 $x = 0, 1, 2$

Find all the zeros of the following polynomials ALGEBRAICALLY

1.  $f(x) = x^3 - x^2 - 10x - 8$  possible zeros  $\pm 1, \pm 2, \pm 4, \pm 8$

4 | 1 -1 -10 -8

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -10 & -8 \\ & \downarrow & 4 & 12 & 8 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

Zeros are -2, -1, 4

$(x^2 + 3x + 2)(x-4) = (x+2)(x+1)(x-4)$

2.  $f(x) = 2x^3 - x^2 - 13x - 6$  possible zeros  $\pm 1, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 3, 6$

-2 | 2 -1 -13 -6

$$\begin{array}{r|rrrr} -2 & 2 & -1 & -13 & -6 \\ & \downarrow & -4 & 10 & 6 \\ \hline & 2 & -5 & -3 & 0 \end{array}$$

$(2x^2 - 5x - 3)(x+2) = (2x+1)(x-3)(x+2)$

3.  $g(x) = x^3 - 9x^2 + 23x - 15$

possible  $\pm 1, 3, 5, 15$

3 | 1 -9 23 -15

$$\begin{array}{r|rrrr} 3 & 1 & -9 & 23 & -15 \\ & \downarrow & 3 & -18 & 15 \\ \hline & 1 & -6 & 5 & 0 \end{array}$$

$(x^2 - 6x + 5)(x-3)$   
 $(x-5)(x-1)(x-3)$

possible  $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, 2, \frac{2}{3}$

4.  $h(x) = 6x^3 - 7x^2 - 9x - 2$

Zero  $-\frac{1}{2}, 3, -2$

2 | 6 -7 -9 -2

$$\begin{array}{r|rrrr} 2 & 6 & -7 & -9 & -2 \\ & \downarrow & 12 & 10 & 2 \\ \hline & 6 & 5 & 1 & 0 \end{array}$$

$(6x^2 + 5x + 1)(x-2)$   
 $(2x+1)(3x+1)(x-2)$

Zeros: 5, 1, 3

Zeros:  $-\frac{1}{2}, -\frac{1}{3}, 2$

possible:  $\pm 1, 2, 3, 6$  4 factors

5.  $g(x) = x^4 - 6x^3 + 11x^2 - 6x$

$x(x^3 - 6x^2 + 11x - 6)$   

$$\begin{array}{r} 2 \overline{) 1 \quad -6 \quad 11 \quad -6} \\ \underline{\phantom{2} 2 \quad -8 \quad 16} \\ 1x^2 - 4x + 3 \quad 0 \end{array}$$

$x(x-2)(x-3)(x+1)$

$x = 0, 2, 3, -1$

6.  $g(x) = x^4 - 5x^2 + 4$  possible  $\pm 1, 2, 4$   
 4 factors

$x^2 - 1$   

$x^4$	$-x^2$
$-4x^2$	$4$

$4x^4 - 4x^2$

$(x^2-1)(x^2-4)$   
 $(x-1)(x+1)(x-2)(x+2)$

8.  $f(x) = x^3 - 4x^2 + 2x + 4$

$$\begin{array}{r} 2 \overline{) 1 \quad -4 \quad 2 \quad 4} \\ \underline{\phantom{2} 2 \quad -4 \quad -4} \\ 1 \quad -2 \quad -2 \quad 0 \end{array}$$

$(x^2-2x-2)(x-2)$

Zero  $x = \pm 1, \pm 2$

possible  $\pm 1, 2, 4$

Zero  $x = 2$

possible  $\pm 1, 2$

7.  $f(x) = x^3 - 4x^2 - 11x + 2$

$$\begin{array}{r} -2 \overline{) 1 \quad -4 \quad -11 \quad 2} \\ \underline{\phantom{-2} -2 \quad 12 \quad -2} \\ 1x^2 - 6x + 1 \quad 0 \end{array}$$

$(x+2)(x^2-6x+1)$

Real Zeros  $x = -2$

17. Identify the zeroes of  $f(x) = (x+3)(x-4)(x-3)$ , write the function in standard form, and state how the zeroes are related to the standard form.

Zeros  $x = -3, 4, 3$

$f(x) = (x^2-9)(x-4)$   
 $= x^3 - 4x^2 - 9x + 36$

Possible The zeros are

$\frac{36}{1} : \pm 1, 2, 3, 4, 6, 9, 12, 18$

19. Explain the Error Sabrina was told to find the zeros of the polynomial function  $h(x) = x(x-4)(x+2)$ . She stated that the zeros of this polynomial are  $x = 0$ ,  $x = -4$ , and  $x = 2$ . Explain her error.

the zeros are  $x = 4$  and  $-2, 0$

She didn't take the opposite of the factor.

Review

Perform the following operations

1.  $(x^2 - 3x + 7) + (3x^2 + 5x - 3)$

$4x^2 + 2x + 4$

2.  $(-3x^2 - 5) - (x^2 + 7x + 12)$

$-3x^2 - 5 - x^2 - 7x - 12$

$-4x^2 - 7x - 17$

3.  $(5x^3 - 1)^2$

$25x^6 - 10x^3 + 1$