

Establish each identity:

1.  $\frac{1 - \cos^2 \theta}{\sin \theta} = \sin \theta$

$$\frac{\sin^2 \theta}{\sin \theta} = \sin \theta$$

2.  $\frac{1 + \tan^2 x}{\csc^2 x} = \tan^2 x$

$$\frac{\sec^2 x}{\csc^2 x} = \frac{1}{\cos^2 x} \cdot \frac{1}{\frac{1}{\sin^2 x}} = \frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1} = \tan^2 x$$

3.  $\frac{\sin^2 u + \tan^2 u + \cos^2 u}{\sec u} = \sec u$

$$\frac{1 + \tan^2 u}{\sec u} = \frac{\sec^2 u}{\sec u} = \sec u$$

4.  $(\sec^2 x + \csc^2 x) - (\tan^2 x + \cot^2 x) = 2$

$$\left( \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} \right) = \frac{1 - \sin^2 x}{\cos^2 x} + \frac{1 - \cos^2 x}{\sin^2 x} = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\sin^2 x} = 1 + 1 = 2$$

5.  $\frac{1}{1 - \cos x} - \frac{1}{1 + \cos x} = 2 \cot x \csc x$

*Common den.*  
 $\frac{1 + \cos x}{1 + \cos x} \cdot \left( \frac{1}{1 - \cos x} \right) - \frac{1 - \cos x}{1 - \cos x} \cdot \left( \frac{1}{1 + \cos x} \right)$   
*distribute*  
 $\frac{1 + \cos x - (1 - \cos x)}{1 - \cos^2 x} = \frac{2 \cos x}{\sin^2 x} = \frac{2 \cos x}{\sin x} \cdot \frac{1}{\sin x} = 2 \cot x \csc x$

6.  $(\sin x)(\tan x + \cot x) = \sec x$

$$\sin x \left( \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) = \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x \cdot \cos x}{\cos x} = \frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$$

7.  $\sin x \cdot \cos x \cdot \tan x \cdot \sec x \cdot \csc x = \tan x$

$$\cancel{\sin x} \cdot \cancel{\cos x} \cdot \frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \frac{1}{\cancel{\cos x}} \cdot \frac{1}{\cancel{\sin x}} = \tan x$$

8.  $\frac{1}{\sin^2 x} + \frac{\sec^2 x}{\tan^2 x} = 2 \csc^2 x$

$$\frac{1}{\sin^2 x} + \frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} = \frac{1}{\sin^2 x} + \frac{1}{\sin^2 x} = \frac{2}{\sin^2 x} = 2 \csc^2 x$$



$$9. \sec(-x)\cos(-x) = 1$$

$$\sec x \cos x = \frac{1}{\cos x} \cdot \cos x = 1$$

$$10. \cot(-x)\tan(-x) = 1$$

$$-\cot(x)(-\tan x) = -\frac{1}{\tan x} \cdot (-\tan x) = 1$$

$$11. \csc \theta \cdot \tan \theta = \sec \theta$$

$$\frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} = \frac{1}{\cos \theta}$$

$$= \sec \theta$$

$$12. \sin \theta (\cot \theta + \tan \theta) = \sec \theta$$

$$\sin \theta \left( \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \right)$$

$$\frac{\cos \theta}{\cos \theta} \cdot \cos \theta + \frac{\sin^2 \theta}{\cos \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta} = \sec \theta$$

$$13. \sin^2 x (1 + \cot^2 x) = 1$$

$$\sin^2 x \left( 1 + \frac{\cos^2 x}{\sin^2 x} \right)$$

$$\sin^2 x + \cos^2 x$$

$$= 1$$

$$14. \tan^2 \alpha \cos^2 \alpha + \cot^2 \alpha \sin^2 \alpha = 1$$

$$\frac{\sin^2 \alpha}{\cos^2 \alpha} \cdot \cos^2 \alpha + \frac{\cos^2 \alpha}{\sin^2 \alpha} \cdot \sin^2 \alpha$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$15. \cot \beta + \tan \beta = \sec \beta \csc \beta$$

$$\frac{\cos \beta}{\sin \beta} + \frac{\sin \beta}{\cos \beta} = \frac{\cos \beta}{\sin \beta} \cdot \frac{\sin \beta}{\cos \beta} + \frac{\sin \beta}{\cos \beta} \cdot \frac{\cos \beta}{\sin \beta}$$

$$\frac{\cos^2 \beta + \sin^2 \beta}{\sin \beta \cos \beta} = \frac{1}{\sin \beta \cos \beta}$$

$$= \sec \beta \csc \beta$$

$$16. \sin^2 \alpha + \tan^2 \alpha + \cos^2 \alpha = \sec^2 \alpha$$

$$1 + \tan^2 \alpha = \sec^2 \alpha$$

$$17. \frac{\sec w}{\csc w} + \frac{\sin w}{\cos w} = 2 \tan w$$

$$\frac{\frac{1}{\cos w}}{\frac{1}{\sin w}} + \frac{\sin w}{\cos w}$$

$$\frac{\sin w}{\cos w} + \frac{\sin w}{\cos w}$$

$$= 2 \frac{\sin w}{\cos w} = 2 \tan w$$

$$18. 1 + \frac{\sin^2 x}{1 + \cos^2 x} = \frac{2}{1 + \cos^2 x}$$

$$\frac{1 + \cos^2 x}{1 + \cos^2 x} + \frac{\sin^2 x}{1 + \cos^2 x}$$

$$\frac{1 + \cos^2 x + \sin^2 x}{1 + \cos^2 x}$$

$$\frac{1 + 1}{1 + \cos^2 x} = \frac{2}{1 + \cos^2 x}$$